PARISHRAM 2024

- FOR 12TH CLASS CBSE BOARD STUDENTS -

- Subject Name- Mathematics
- Chapter Name- Relations and Functions

Lecture No.- 06

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Today's Targets

1. One-One Function
2. Onto Function
3. ----------
4. ----------

Question

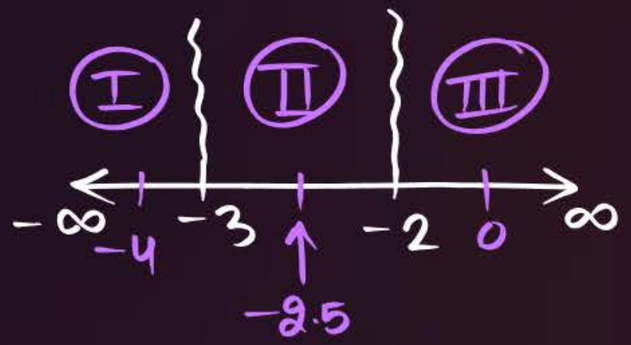
Find the domain of the following functions

i) $f(x) = \sqrt{9 - x^2}$

ii) $f(x) = \dfrac{x-3}{x+4}$

iii) $f(x) = \dfrac{2x+3}{x^2-3x+2}$

iv) $f(x) = \dfrac{1}{\sqrt{x^2+5x+6}}$



$$f(x) = \frac{1}{\sqrt{x^2+5x+6}}$$

$$x^2+5x+6 > 0$$

$$(x+3)(x+2) > 0$$

$x = -3$ $\quad$ $x = -2$

Int.	$(x+3)(x+2)$	Sign
$(-\infty,-3)$	$(-1)(-2)$	+
$(-3,-2)$	$(0.5)(-0.5)$	−
$(-2,\infty)$	$(3)(2)$	+

$$x \in (-\infty,-3) \cup (-2,\infty)$$

Range

① If $y = f(x)$, then derive an expression $x = g(y)$ using $y = f(x)$

② Use the restrictions of domain in y.

Domain

① $\frac{1}{\text{Deno} \neq 0}$ ② $\sqrt{\text{Non-negative}}$

Question

Find the range of the following functions

i) $f(x) = 5 - 4x$

ii) $f(x) = x^2 + 4$

iii) $f(x) = x^3 + 1$

iv) $f(x) = x^2 + 4x + 5$

(1) $y = 5 - 4x$

$4x = 5 - y \Rightarrow \boxed{x = \frac{5-y}{4}}$

$\hookrightarrow y \in R$

$\Rightarrow$ Range $= (-\infty, \infty)$

Alternate Method.

$f(x) = 5 - 4x$

$\hookrightarrow x \in (-\infty, \infty)$

$-\infty < x < \infty$

$\infty > -4x > -\infty$

$\infty > 5 - 4x > -\infty \Rightarrow -\infty < f(x) < \infty$

Question

Find the range of the following functions

i) $f(x) = 5 - 4x$

ii) $f(x) = x^2 + 4$

iii) $f(x) = x^3 + 1$

iv) $f(x) = x^2 + 4x + 5$

② $y = x^2 + 4 \Rightarrow x^2 = y - 4 \Rightarrow x = \pm\sqrt{y-4}$

$y - 4 \geq 0$

$\Rightarrow y \geq 4$

$\Rightarrow$ Range $= [4, \infty)$

Alternate Method

$f(x) = x^2 + 4 \Rightarrow x \in R$

$-\infty < x < \infty$

$0 \leq x^2 < \infty$

$0 + 4 \leq x^2 + 4 < \infty$

$4 \leq f(x) < \infty$

Range $= [4, \infty)$

Question

Find the range of the following functions

i) $f(x) = 5 - 4x$

ii) $f(x) = x^2 + 4$

iii) $f(x) = x^3 + 1$

iv) $f(x) = x^2 + 4x + 5$

③ $y = x^3 + 1 \Rightarrow x^3 = y - 1 \Rightarrow x = (y-1)^{1/3}$

$y \in R$

Range $= (-\infty, \infty)$

Alternate Method

$f(x) = x^3 + 1 \Rightarrow x \in R$

$-\infty < x < \infty$

$-\infty < x^3 < \infty$

$-\infty + 1 < x^3 + 1 < \infty$

$-\infty < f(x) < \infty$

Range $= (-\infty, \infty)$

Question

Find the range of the following functions

i) $f(x) = 5 - 4x$

ii) $f(x) = x^2 + 4$

iii) $f(x) = x^3 + 1$

iv) $f(x) = x^2 + 4x + 5$

(4) $y = x^2+4x+5 \Rightarrow y = (x+2)^2-(2)^2+5$

$\Rightarrow y = (x+2)^2+1$

$\Rightarrow (x+2)^2 = y-1$

$\Rightarrow x+2 = \pm\sqrt{y-1}$

$\Rightarrow x = -2 \pm \sqrt{y-1}$

$y-1 \geq 0$

$y \geq 1$

Range $= [1, \infty)$

Alternate Method.

$f(x) = x^2+4x+5$

$= (x+2)^2-(2)^2+5$

$f(x) = (x+2)^2+1$

we know, $0 \leq (x+2)^2 < \infty$

$1 \leq (x+2)^2+1 < \infty$

$1 \leq f(x) < \infty \Rightarrow$ Range $= [1, \infty)$

Homework

Find the domain of the function $f(x) = \sqrt{x^2 + 3x + 2}$.

A $(-\infty, -2) \cup (-1, \infty)$

B $(-\infty, -2] \cup [-1, \infty)$

C $(-2, -1)$

D $[-2, -1]$

$x^2+3x+2 \geq 0$

$(x+1)(x+2) \geq 0$

Int	$(x+1)(x+2)$	Sign
$(-\infty,-2)$		+
$(-2,-1)$		−
$(-1,\infty)$		+

$x \in (-\infty,-2] \cup [-1,\infty)$

Homework

Let $A = \{7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$ and let $f: A \to N$ be defined by $f(x) =$ the highest prime factor of x. How many elements are there in the range of f?

A. 4

B. 5

C. 6 ✓

D. 7

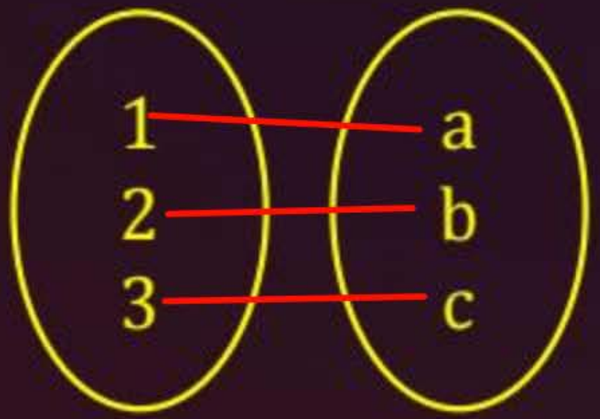
x	$f(x)$
$7 = 7$	7
$8 = 2\times2\times2$	2
$9 = 3\times3$	3
$10 = 5\times2$	5
$11 = 11$	11
$12 = 2\times2\times3$	3
$13 = 13$	13
$14 = 2\times7$	7
$15 = 3\times5$	5
$16 = 2\times2\times2\times2$	2

Range $= \{7, 2, 3, 5, 11, 13\}$

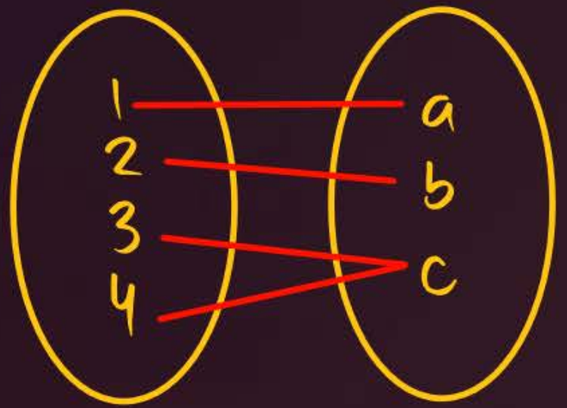
Function (Class XI)

Every element in domain must have **one** and **only one image** in co-domain

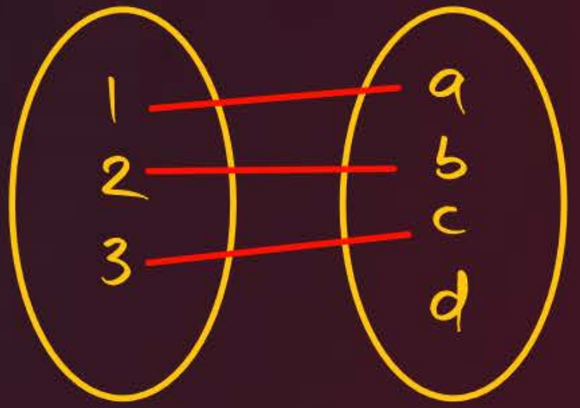
$n(A) \geq n(B)$



function



function



function

Types of functions

- One-one function
- Onto function
- Bijective function
- Even function
- odd function

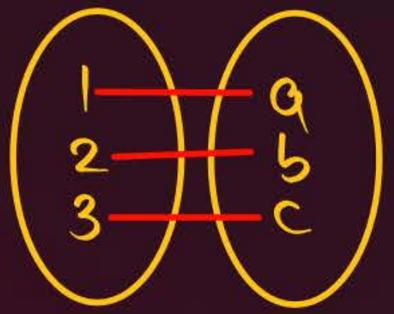
One-One Function

One-one function is also called injective function

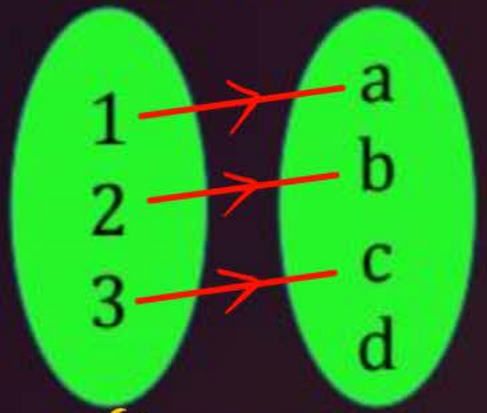
One-One Function

- Every $x \in A$ must have unique image
- $n(A) \leq n(B)$
- Range $\subseteq$ Co-domain

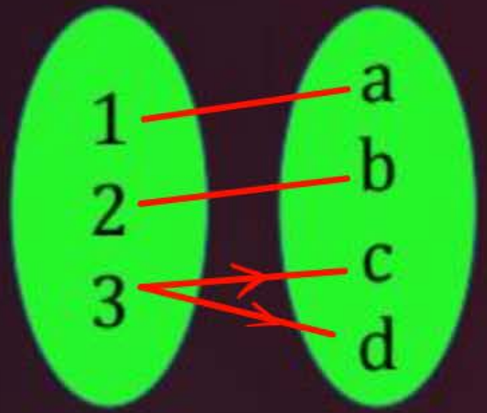
$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$



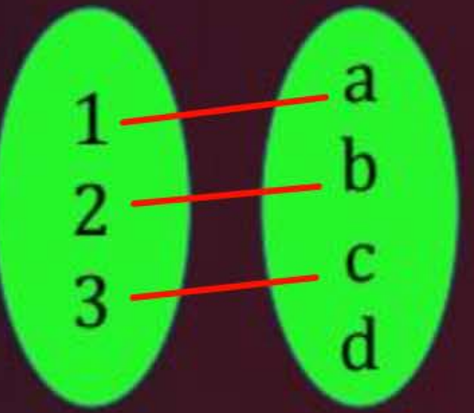
one-one ✓



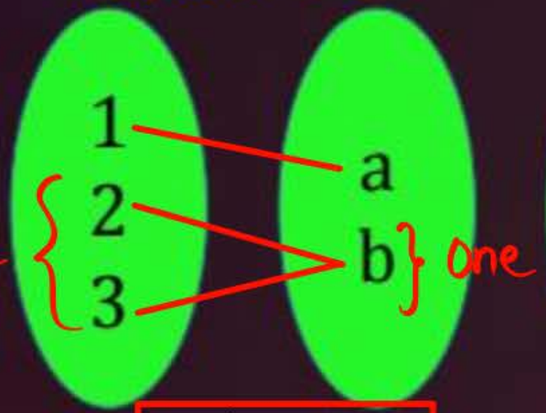
Not function

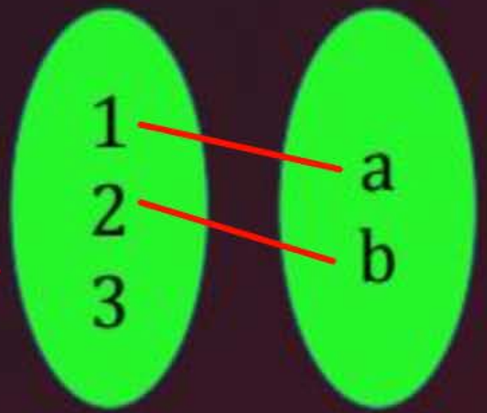


one-one function



Not one-one

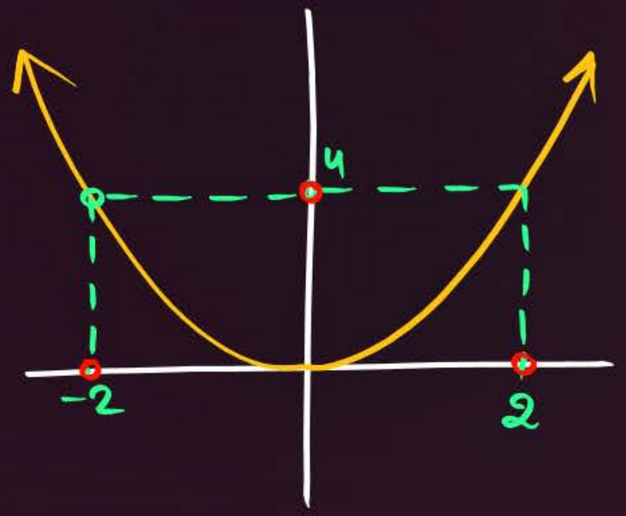




If a function is not one-one then it is called Many one function.

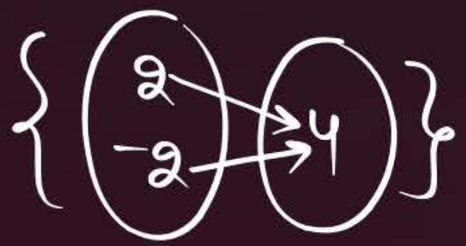
$n(A) \geq n(B)$ → One-One function (Not Possible)

$f(x) = x^2$

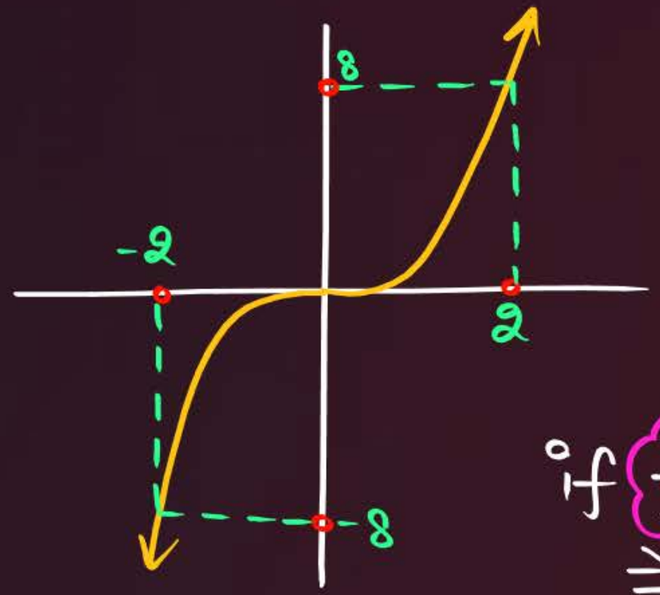


$f(2) = f(-2)$

But $2 \neq -2$



$f(x) = x^3$



°if $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

① $f(x) = x^2$

* One-One ness

Let $f(a) = f(b)$

$a^2 = b^2$

$a^2 - b^2 = 0$

$(a+b)(a-b) = 0$

$a+b=0$ or $a-b=0$

$a=-b$ $\quad$ $a=b$

$\Rightarrow$ f is not one-one

② $f(x) = x^3$

* One-one ness.

Let $f(a) = f(b)$

$a^3 = b^3$

$a^3 - b^3 = 0$

$(a-b)(a^2+ab+b^2) = 0$

↳ is never zero. for $a, b \neq 0$

$(a-b) = 0$

$a=b$

$\Rightarrow$ f is one-one.

Onto Function

Onto function is also called surjective function

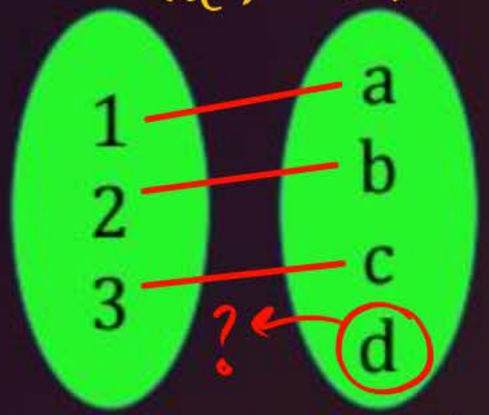
Onto Function

- Every $y \in B$ must have at least one pre-image in A
- $n(B) \leq n(A)$
- Range = Co-domain

$$\forall\, y \in B\ \exists\, x \in A : y = f(x)$$

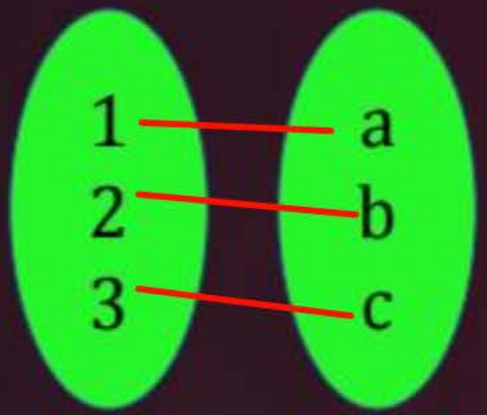
at least one → कम से कम 1

$n(A) < n(B)$



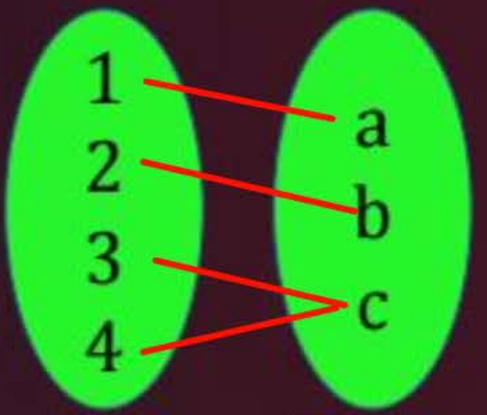
one-one ✓
onto ✗

$n(A) = n(B)$



one-one ✓
onto ✓

$n(A) > n(B)$

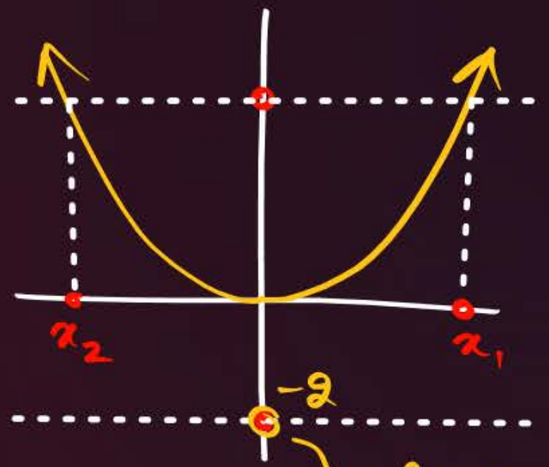


one-one ✗
onto ✓

* If a function is not onto then it is called into.

① $f: R \to R$ (codomain)

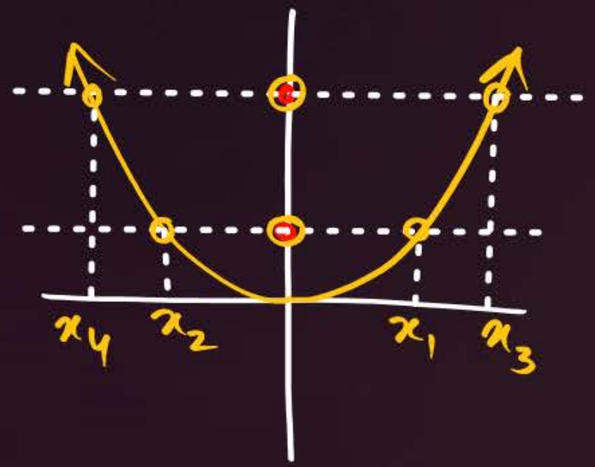
$f(x) = x^2$



for $y = -2$ there is no such $x \in R$ such that $x^2 = -2$

② $f: R \to [0, \infty)$ (codomain)

$f(x) = x^2$



Onto

for every $y \in B$ there exist at least one $x \in A$ such that $y = f(x)$

THANK
YOU